MI0213
Dynamic Aperture for the Recycler Lattices RRv16 and RRv17

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Introduction

The dynamic aperture is often used as a measure of the quality of an accelerator lattice. If the dynamic aperture is larger than the physical aperture, or is at least much larger than the emittance of the beam in the accelerator, then the lattice design is considered acceptable. In the case of the Recycler, the larger the dynamic aperture, and hence the emittance of the stored beam, the greater will be the contribution the Recycler will make to increasing the luminosity of the Tevatron Collider because of the larger number of $\overline{p}$ stored.

As usually calculated, and expressed, the dynamic aperture is the largest emittance of a particle which a tracking code can track, i.e. is stable, for some large number of turns, often 10^5 or 10^6 [1]. There are several difficulties with this methodology. The first problem is due to the fact that the number of turns that a particle can be realistically tracked is far smaller than the number of turns for which we want to be confident of the stability of the beam. In the Recycler 10^5 turns corresponds to about $1s$ of beam time and we want the beam to be stable in the physical lattice for more than $10^4 s$.

The concept of emittance is derived from the invariants of a linear lattice. The finite value for the dynamic aperture is a reflection of the non-linearities in the lattice. As will be shown, particles with the same value of the (linear)

[1] The number of turns for which one track is generally determined by one's patience and the available computer time.

emittance, but with different initial coordinates on the linear phase ellipse, can survive for different numbers of turns.

Another problem is that there are two different emittances, ϵ_h and ϵ_v, in the transverse phase space. The number of turns for which a particle survives will depend on both ϵ_h and ϵ_v making it even more problematical to describe the dynamic aperture in terms of one number. While it may be true, due to coupling, that the average emittances of the ensemble of particles that constitute the beam is the same in the horizontal and vertical planes, the individual particles will have different emittances in the two planes, and it is the survivability of the individual particles that determines the quality of the lattice.

This note will describe calculations made of the dynamic aperture of two versions of the Recycler lattice RRv16 and RRv17 [2]. The description of the lattice was converted from MAD (or TRANSPORT format) into the format used by TEVLAT [3], which was used for the tracking. A comparison of the calculations in MAD and TEVLAT reveals no difference in the calculations of the linear lattice functions and only a small difference in the calculation of the chromaticity [4].

The only non-linearities in the lattices are the sextupole components of the fields in the lattice bend magnets, i.e. I am not considering the high order moments due to magnet field errors.

Calculated Dynamic Aperture

Figure 1 shows the values for the emittance for RRv16 in the case where $\epsilon_x = \epsilon_y$ and $(\beta_x \cdot x' + \alpha_x \cdot x) = (\beta_y \cdot y' + \alpha_y \cdot y) = 0$ in which a particle completes for 1024 turns. [5]

The existence of an "island", a stable region for the particle beyond the

[2]I want to thank Dave Johnson for providing us with the file and other help that he has given me.

[3]A. Russell; private communication.

[4]The difference is ≈ 1 unit of chromaticity unit out of a change of ≈ 30 units from the sextupoles.

[5]In this figure, and in plots of the dynamic aperture that follow, an X represents a stable particle, a square, open or filled represents an unstable particle. An open square is used when the particle is launched at more than one value of the phase on the linear phase ellipse and the particle is not stable for all the configurations. If there is an X in the open square then some of the initial configurations were stable.

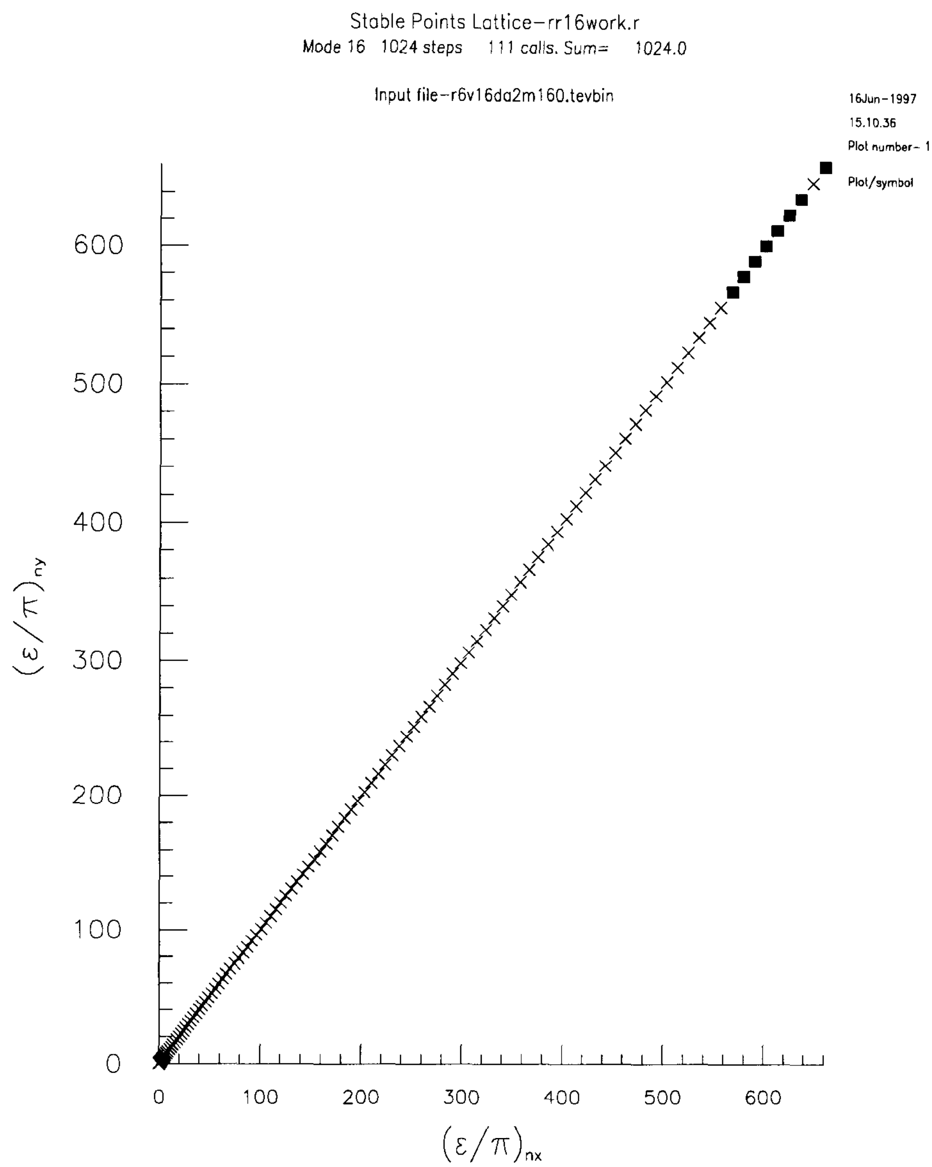

Figure 1: Survival of Particles for 1,024 Turns, $\epsilon_x = \epsilon_y$ and $(\beta_x \cdot x' + \alpha_x \cdot x) = (\beta_y \cdot y' + \alpha_y \cdot y) = 0$.

point where a particle was unstable is clearly seen in the figure. The existence of islands introduces an additional uncertainty into the calculation of the dynamic aperture if the methodology used involves reducing the amplitude of the particle until it proves stable in the tracking and then stopping.

Increasing the number of turns to 10^4 (Figure 2) and, tracking over a finer grid, does not eliminate the island. The use of a finer grid reveals the existence of still other islands. Increasing the number of turns to 10^5 (Figure 3) still shows islands, though somewhat smaller than before. The dynamic aperture, that is the maximum stable emittance before encountering an unstable region as a function of the number of turns is shown in table I.

TABLE I
THE DEPENDENCE OF THE DYNAMIC APERTURE FOR RRv16 AS A FUNCTION OF THE NUMBER OF TURNS TRACKED.

Number of turns.	Maximum Emittance ($mmmr$)
1,024	556π
10,240	496π
102,400	492π

The emittance is a weak function of the number of turns when the number of turns is over 10^4 and even tracking for only 10^3 turns gives a reasonable estimate of the dynamic aperture, when compared to the results computed by tracking for a large number of turns. As we will see, these calculated values of dynamic aperture are overestimates when we do the tracking with additional constraints.

All the tracking described above was done by starting at the same phase in (x, x') and (y, y') phase space. Tracking was next done starting at 3 different values of the initial phase in each plane, and tracking for 2,048 turns. (Figure 4). The squares with an x represent values of the linear emittance where some, but not all, of the initial configurations were stable. In this case the maximum emittance for a particle which was stable for all the initial starting locations in phase space is only $\epsilon_x = \epsilon_y \approx 310\pi mmmr$ [6].

[6] As has been emphasized by L. Michelotti, the linear invariant is not a true invariant of the system and it is therefore not surprising that starting at different values on the phase ellipse yields different values for the dynamic aperture. The dynamic aperture expressed in terms of a linear invariant is at best a crude measure of the stable region of the beam in phase space.

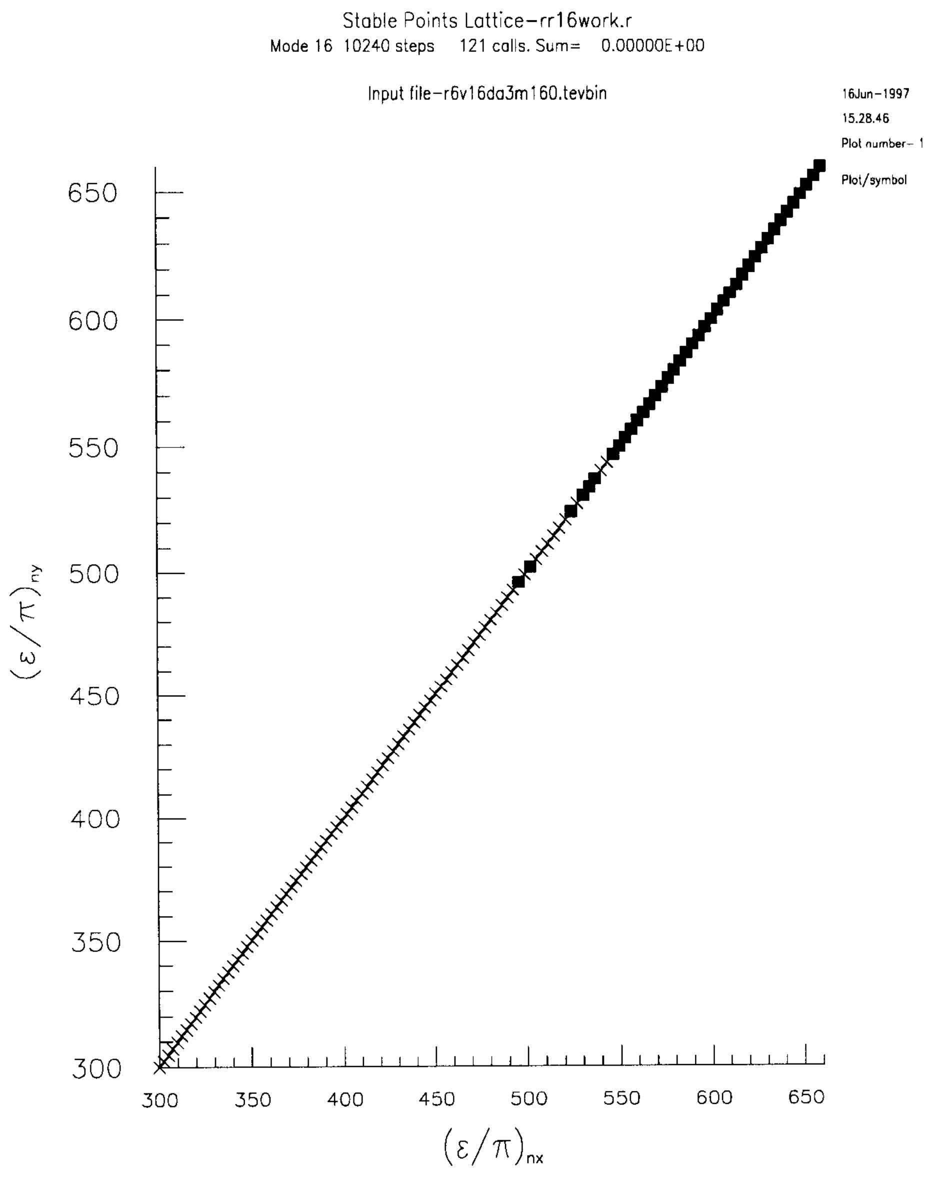

Figure 2: Survival of Particles for 10,240 Turns, $\epsilon_x = \epsilon_y$ and $(\beta_x \cdot x' + \alpha_x \cdot x) = (\beta_y \cdot y' + \alpha_y \cdot y) = 0$.

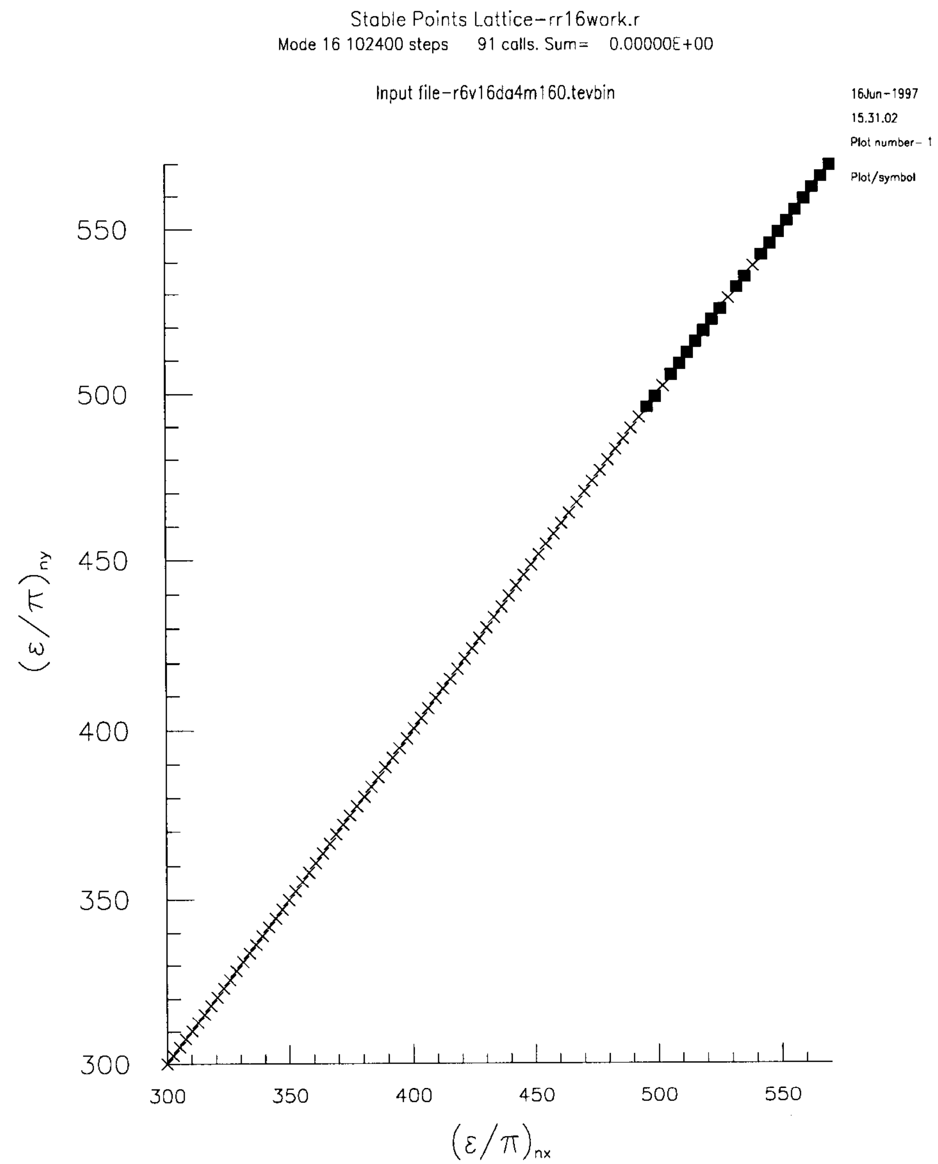

Figure 3: Survival of Particles for 102,400 Turns, $\epsilon_x = \epsilon_y$ and $(\beta_x \cdot x' + \alpha_x \cdot x) = (\beta_y \cdot y' + \alpha_y \cdot y) = 0$.

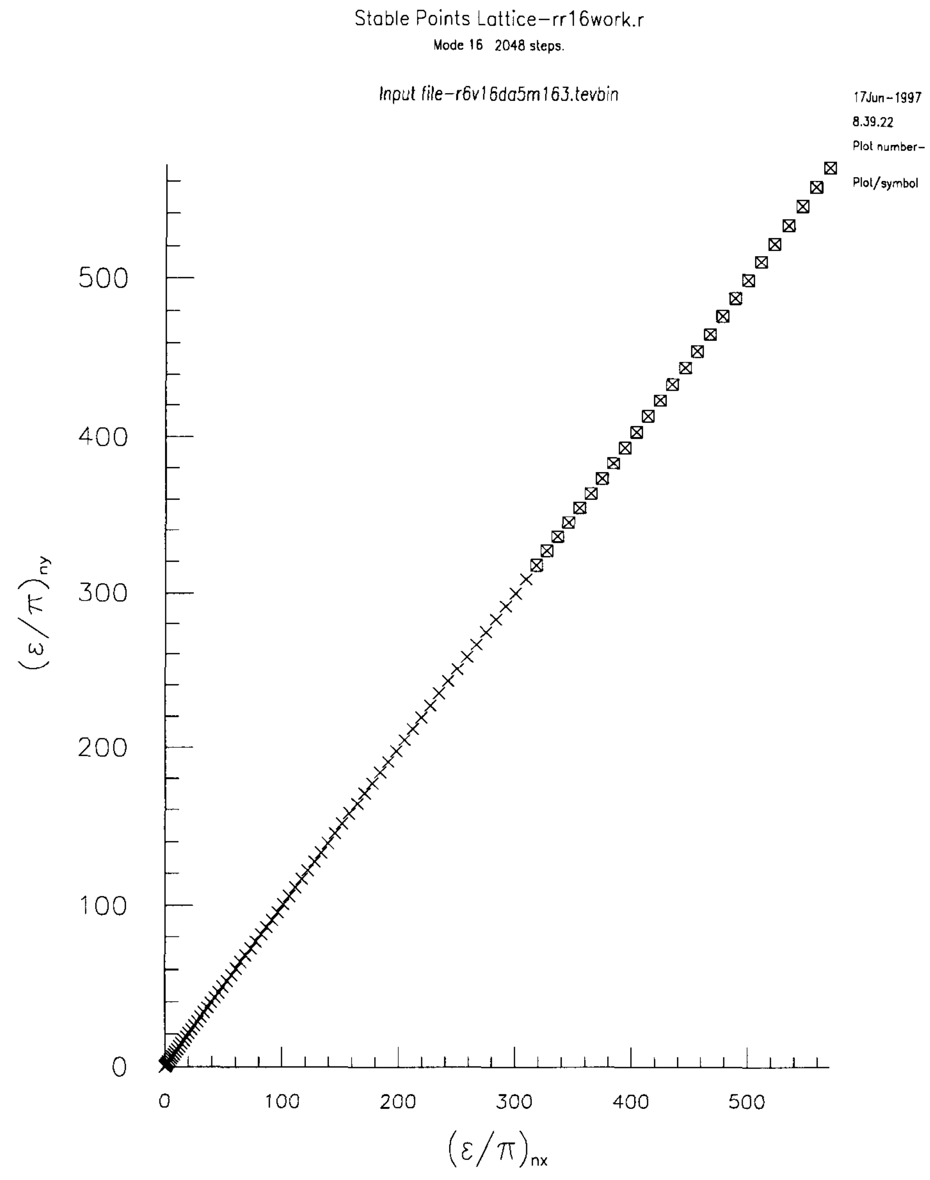

Figure 4: Survival of Particles for 2,048 Turns, $\epsilon_x = \epsilon_y$ and 3x3 values of $(x, x'); (y, y')$.

This is significantly smaller than the values in Table I. Some of the points seen as stable in Figure 3 are points no longer regarded as stable when we launch the particle at different points on the linear phase ellipse.

The tracking described above was repeated for lattice RRv17 (Figures 5, 6 and 7.) The calculated apertures are shown in Table II.

TABLE II
THE DEPENDENCE OF THE DYNAMIC APERTURE FOR RRv17 AS A FUNCTION OF THE NUMBER OF TURNS TRACKED

Number of turns	Maximum Emittance $mmmr$
1,024	513π
10,240	483π
102,400	446π

Requiring that the particle be stable for 2,048 turns, and starting at three different initial positions on the each linear phase ellipse, (Figure 8) yields a dynamic aperture of $\approx 222\pi mmmr$.

The calculated dynamic aperture for RRv17 is similar to, and about 10% smaller than that calculated for RRv16 except when we start at different values of the phase ellipse. In that case there is a large difference in what I would call the dynamic aperture. The challenge is to understand the reason for this difference.

Tune Variation With Amplitude

The tune shift is an involved function not only of the initial amplitudes in the x and y planes but also depends on the initial phase in phase space. In the interests of clarity of presentation, and to simplify the situation, we will look separately at the tunes as a function of the initial x, initial y, and the case where $\epsilon_x = \epsilon_y$ and with $\alpha_x \cdot x + \beta_x \cdot x' = 0$, to see if we can understand the difference in the dynamic aperture.

For the simplest case, in which the displacement is along the x axis, the phase space plots (RRv16; Figure 9, RRv17; Figure 10) for a particle starting near the edge of the dynamic aperture show a pattern characteristic of resonance loss for both RRv16 and RRv17 [7]. This suggests that we look at the tunes as a clue to understanding the dynamic aperture.

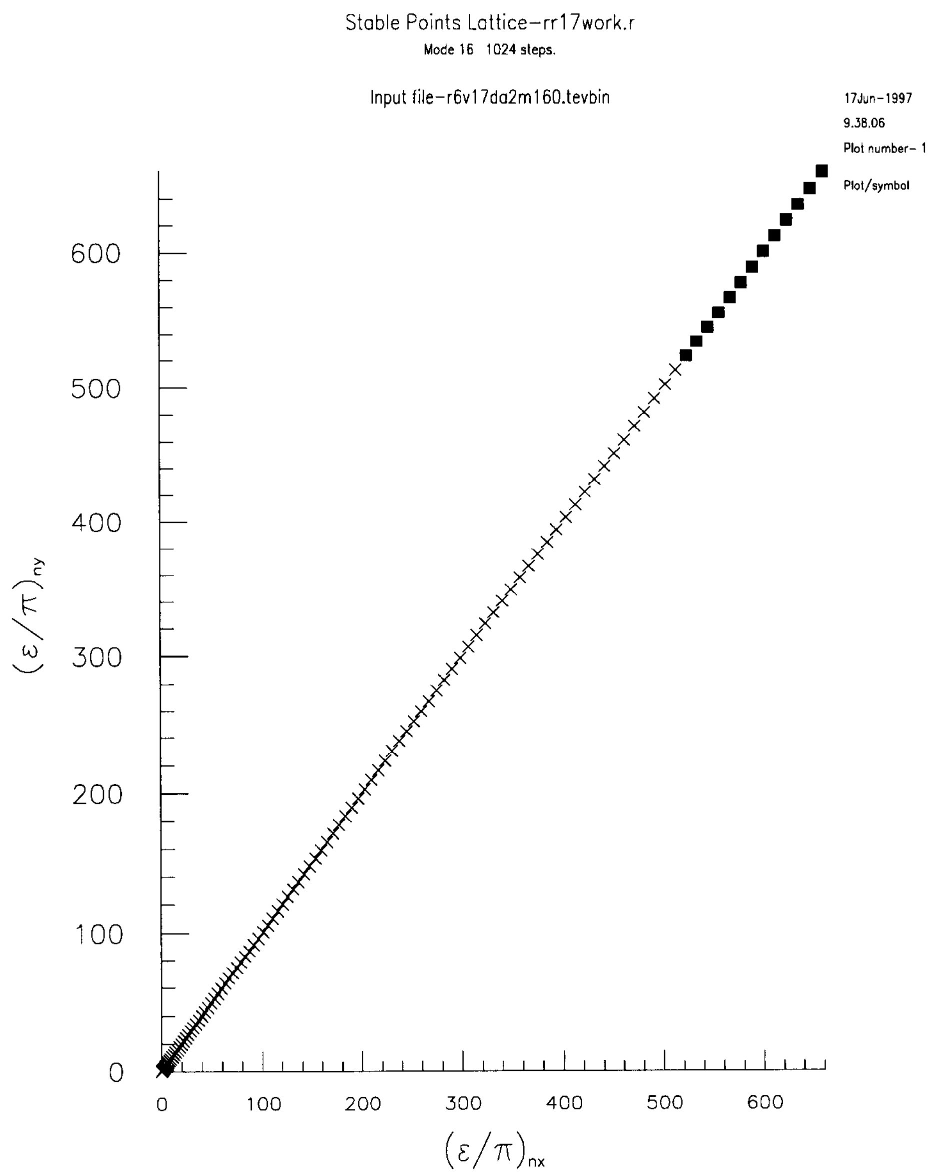

Figure 5: Survival of Particles for 1,024 Turns, $\epsilon_x = \epsilon_y$ and $(\beta_x \cdot x' + \alpha_x \cdot x) = (\beta_y \cdot y' + \alpha_y \cdot y) = 0$.

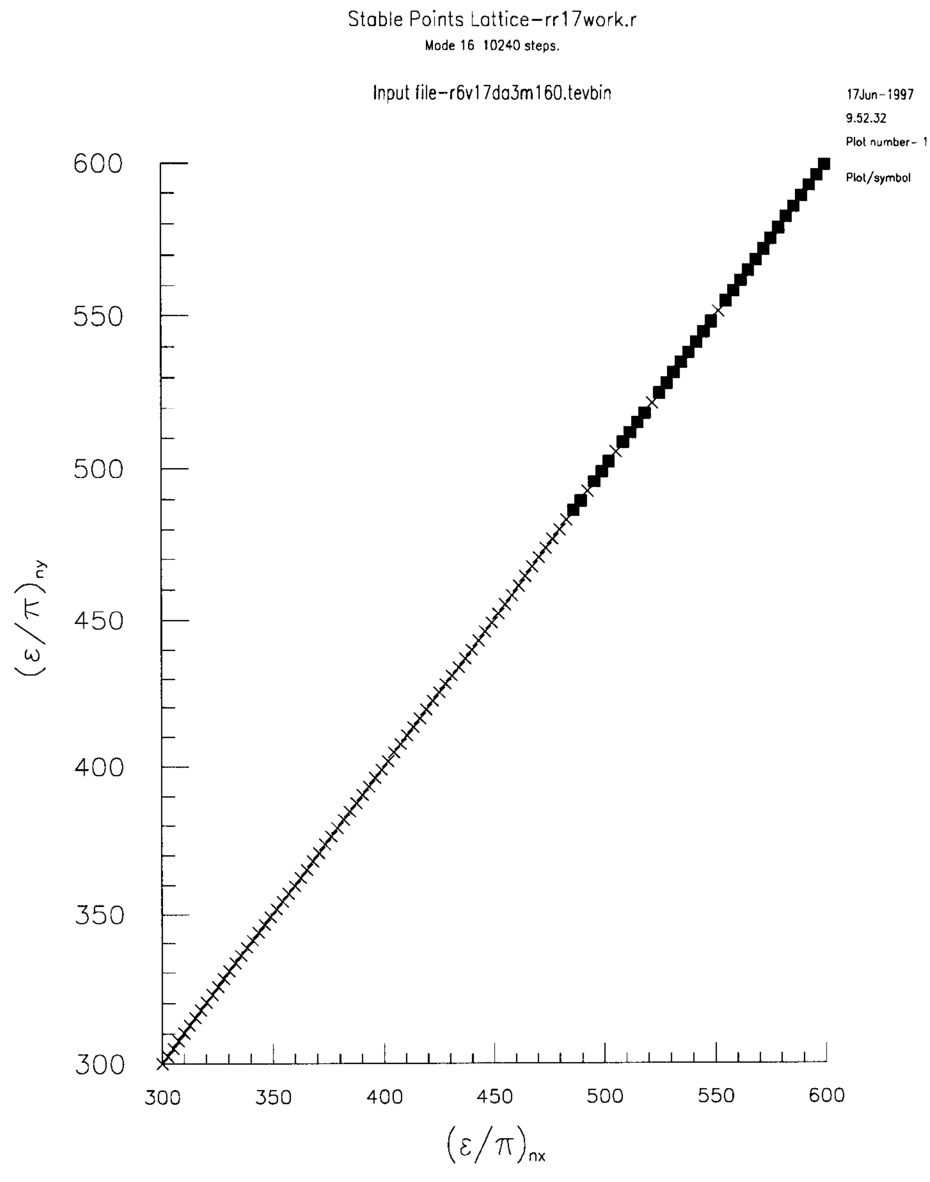

Figure 6: Survival of Particles for 10,240 Turns, $\epsilon_x = \epsilon_y$ and $(\beta_x \cdot x' + \alpha_x \cdot x) = (\beta_y \cdot y' + \alpha_y \cdot y) = 0$.

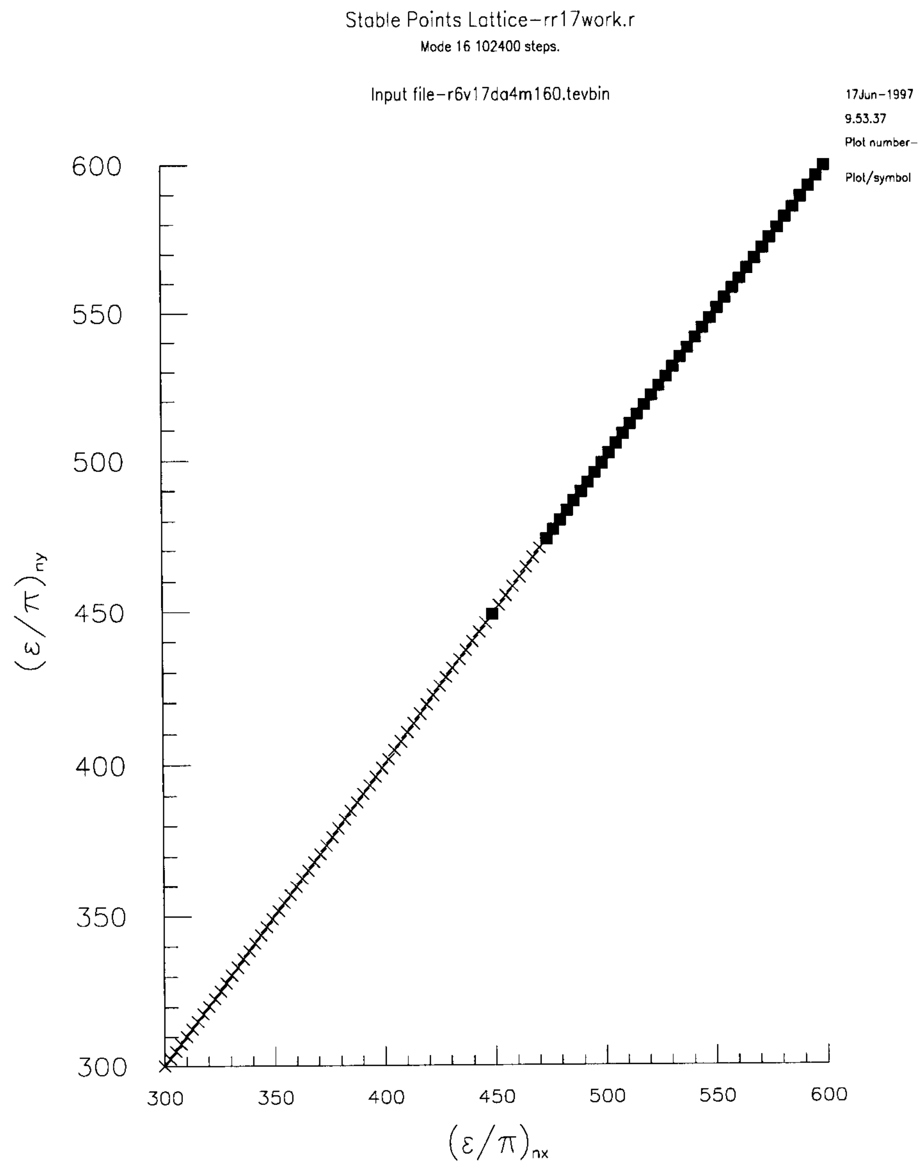

Figure 7: Survival of Particles for 102,400 Turns, $\epsilon_x = \epsilon_y$ and $(\beta_x \cdot x' + \alpha_x \cdot x) = (\beta_y \cdot y' + \alpha_y \cdot y) = 0$.

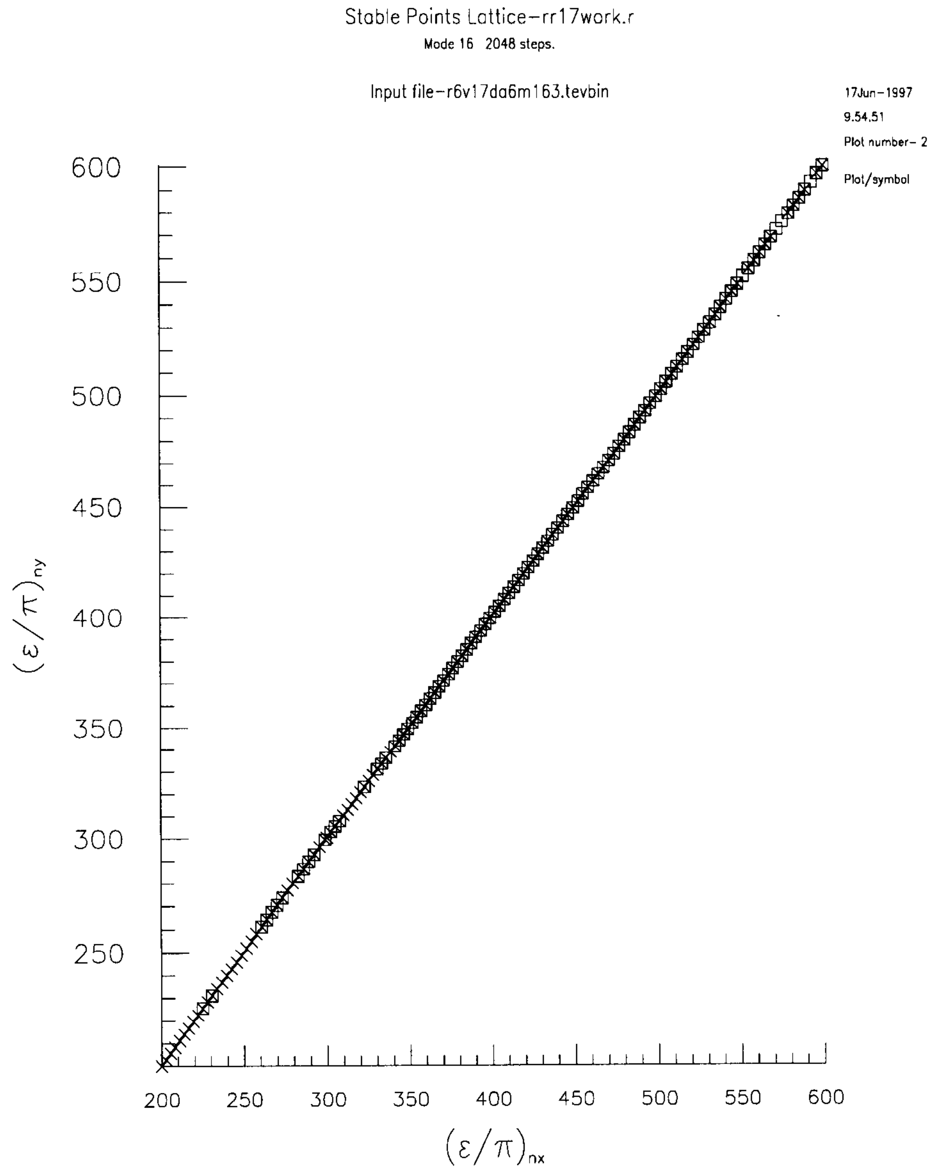

Figure 8: Survival of Particles for 2,048 Turns, $\epsilon_x = \epsilon_y$ and 3x3 values of $(x, x'); (y, y')$.

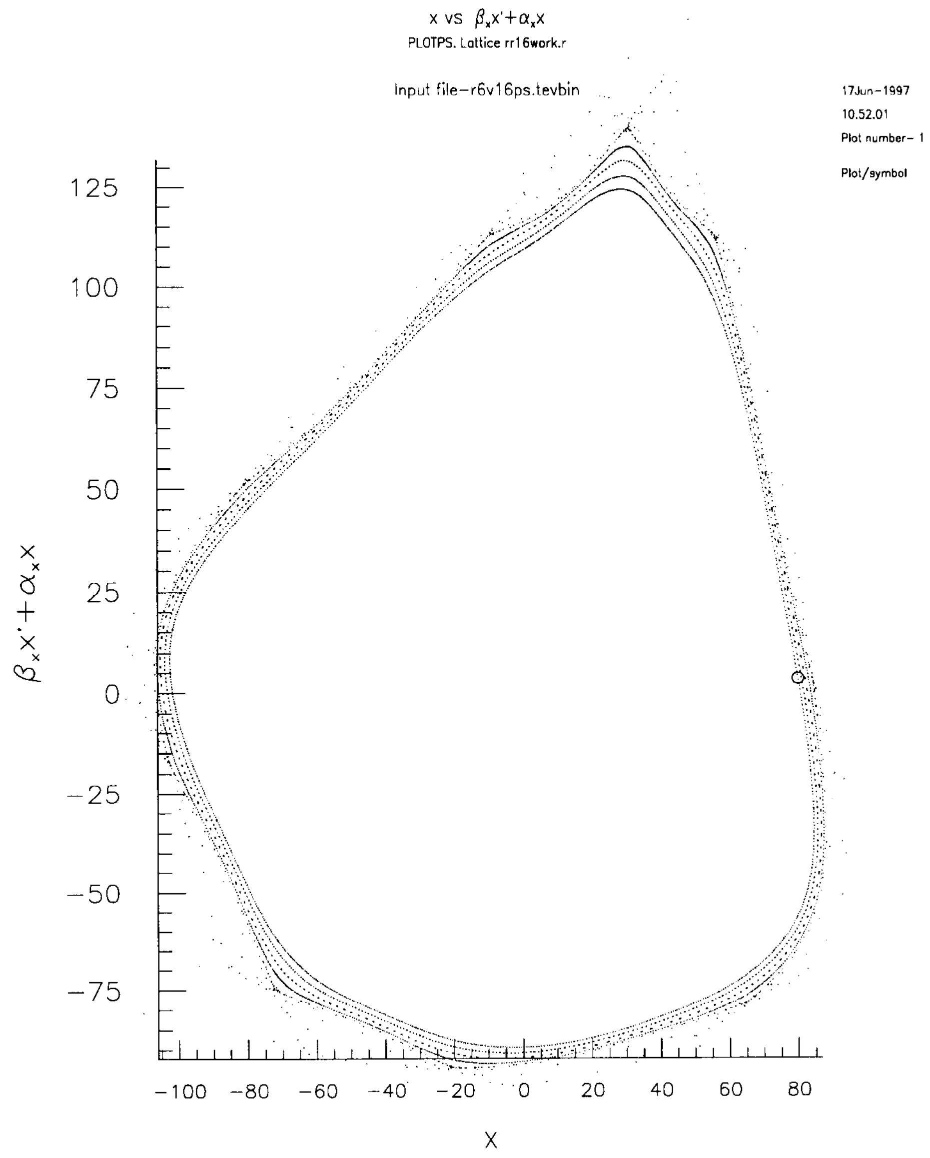

Figure 9: Phase Space Distribution for a Particle in the (x, x') Plane.

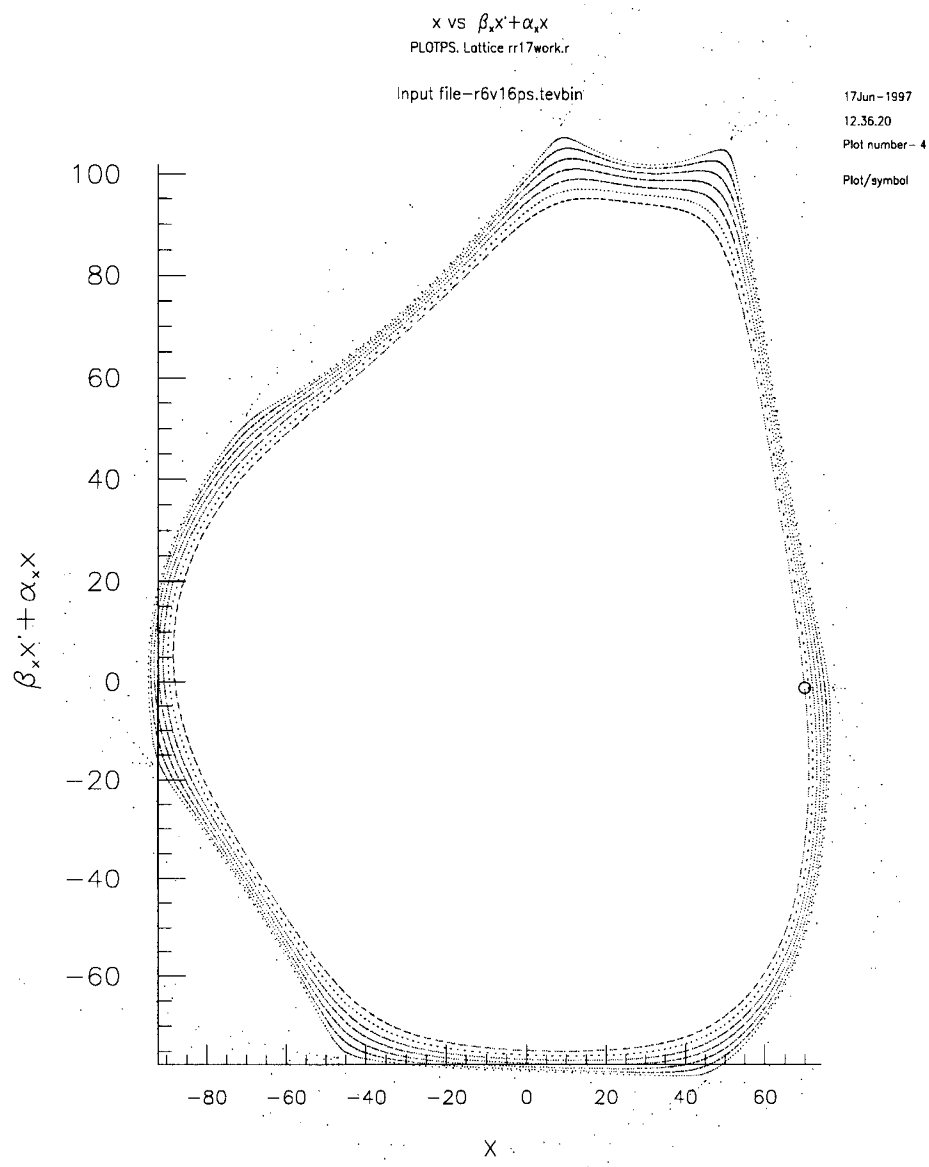

Figure 10: Phase Space Distribution for a Particle in the (x, x') Plane.

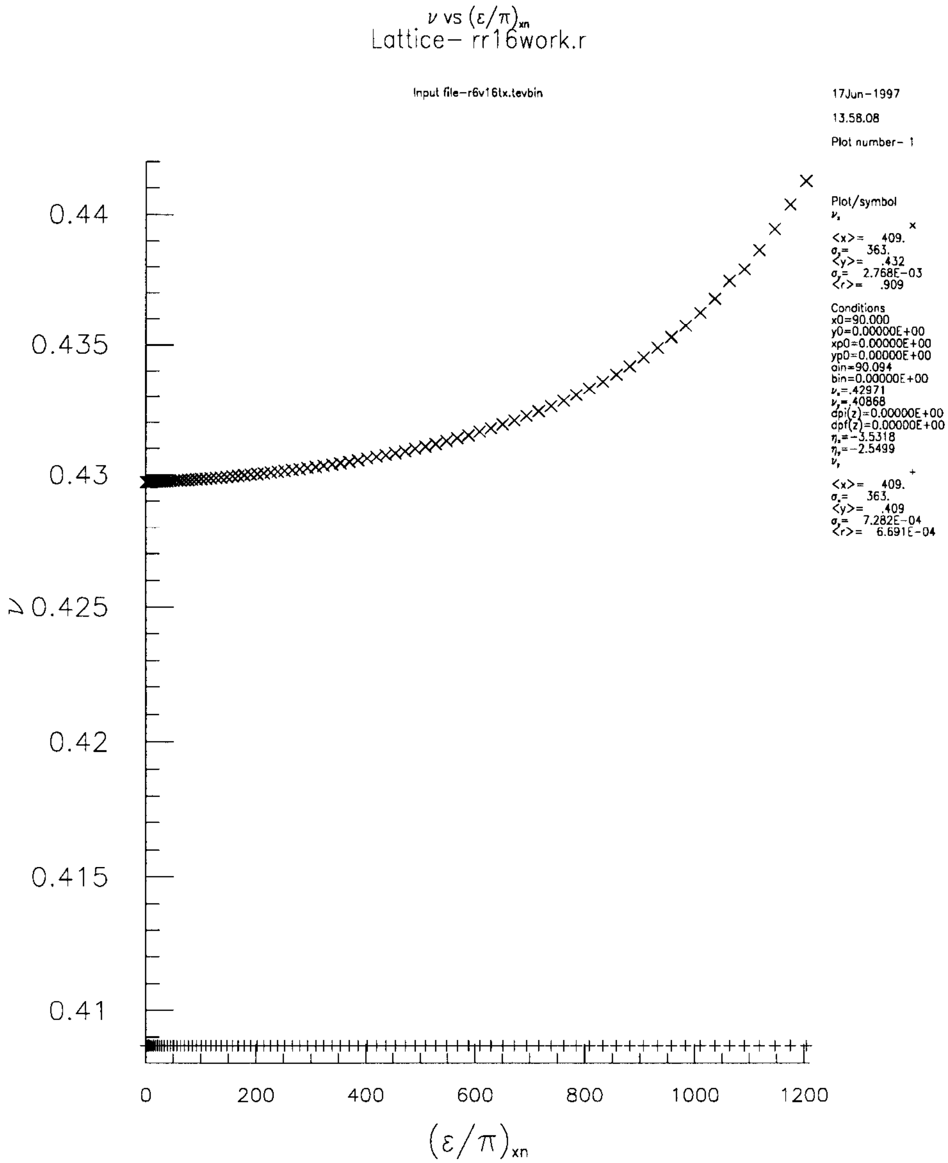

Figure 11: Tune as a function of ϵ_x; Particle started with $x' = y = y' = 0$.

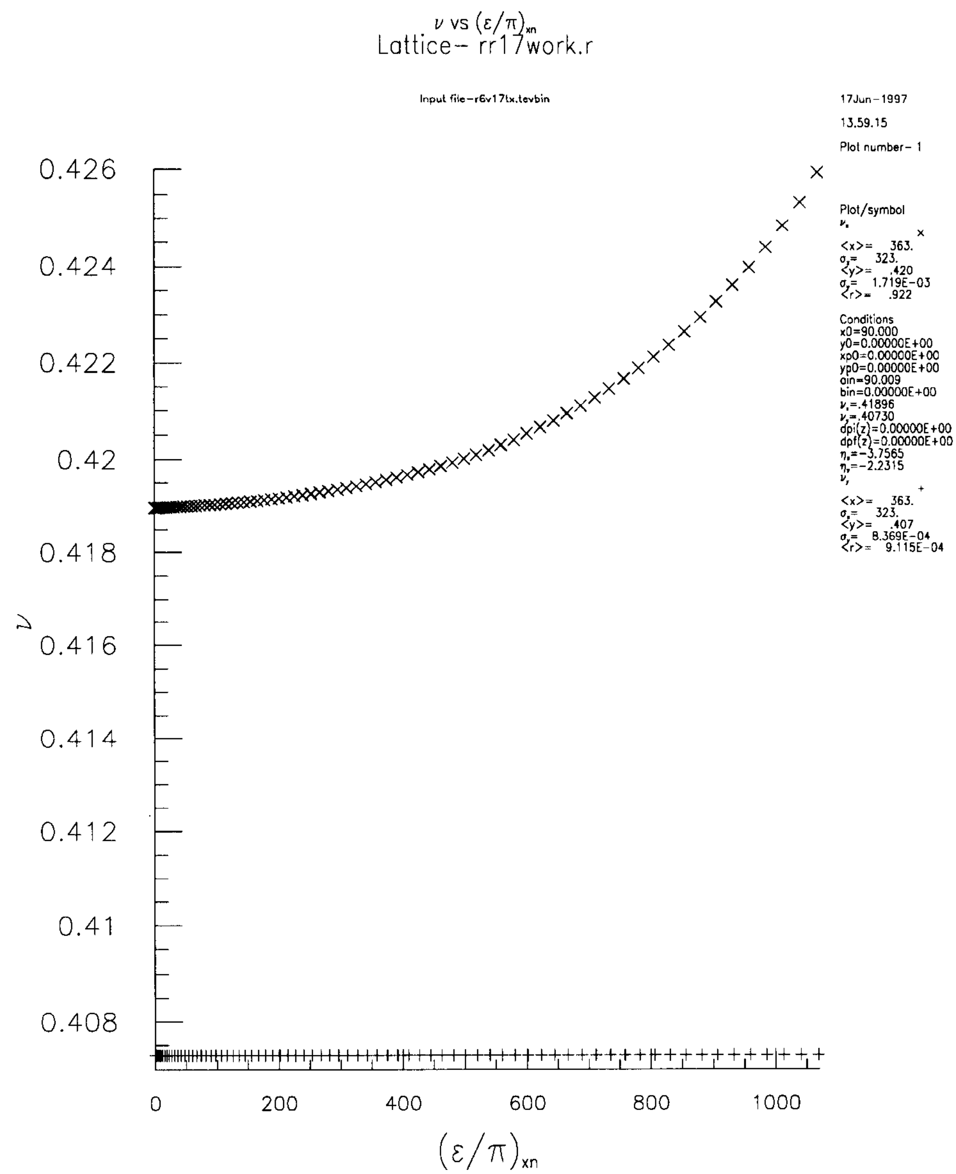

Figure 12: Tune as a function of ϵ_x; Particle started with $x' = y = y' = 0$.

The variation of ν_x with amplitude is small (Figures 11 and 12). Tracking for 2,048 turns, the motion appears stable out to an emittance $\epsilon_x \approx 1200\pi mmmr$ for RRv16 and $\epsilon_x \approx 1070\pi mmmr$ for RRv17[8]. Near the edge of the stable region the phase space of the particle motion shows the characteristics of resonant behavior (Figures 9 and 10). On the other hand the value of the tune, as determined by an fft on the turn by turn results of the tracking, at which the particle is lost is different for the two versions of the lattice. It is not obvious from either Figure 9 or 10 what is the value of the resonant tune.

In the case of an initial displacement in the y direction the motion is more complicated. Due to the coupling from the sextupoles, we have motion in both planes. The dynamic aperture (determined by tracking for 2,048 turns) is $\epsilon_y \approx 738\pi mmmr$ for RRv16 and $\epsilon_y \approx 1100\pi mmmr$ RRv17. The change in ν_y is small (Figures 13 and 14) but we see a rapid decrease in ν_x as ϵ_y is increased. What is striking, in both cases, is the behavior of the tunes as ν_x approaches ν_y. We get a large amplitude in the (x, x') plane, and the size of this x amplitude exhibits a large variation over a couple of hundred turns. On the other hand I have not seen long term growth of the amplitude and I see no suggestion that the particle will become unstable, though that is a possibility if one wanted to track for enough turns. Furthermore the fft of the tracking data in this region shows not a spectrum with just a few lines, but a spectrum rich with many lines, many of which are in the region of the dominant tune. The nature of the spectrum makes the assignment of a tune difficult. This is most likely a consequence of the rapid variation of the amplitudes due to the coupling and the amplitude dependence of the tunes. This behavior is, in my opinion, worthy of additional analytic study.

The final case considered is when $\epsilon_x = \epsilon_y$. In this case the dynamic aperture is apparently $\epsilon_x = \epsilon_y \approx 525\pi mmmr$ for RRv16 and only $\approx 507\pi mmmr$ for RRv17. These values are about what one would expect from Table I and Table II based on the number of turns. The tunes in both planes are rapidly varying as the emittance is varied with the change in ν_y greater than the change in ν_x (Figures 15 and 16). The behavior is smooth over most of the interval studied but there are regions where the dependence is irregular.

[7] L. Michelotti, Intermediate Classical Dynamics with Applications to Beam Physics , Wiley Interscience 1995

[8] Please note that the emittance of a stable particle launched along the x axis can be greater than a particle launched with $\epsilon_x = \epsilon_y$.

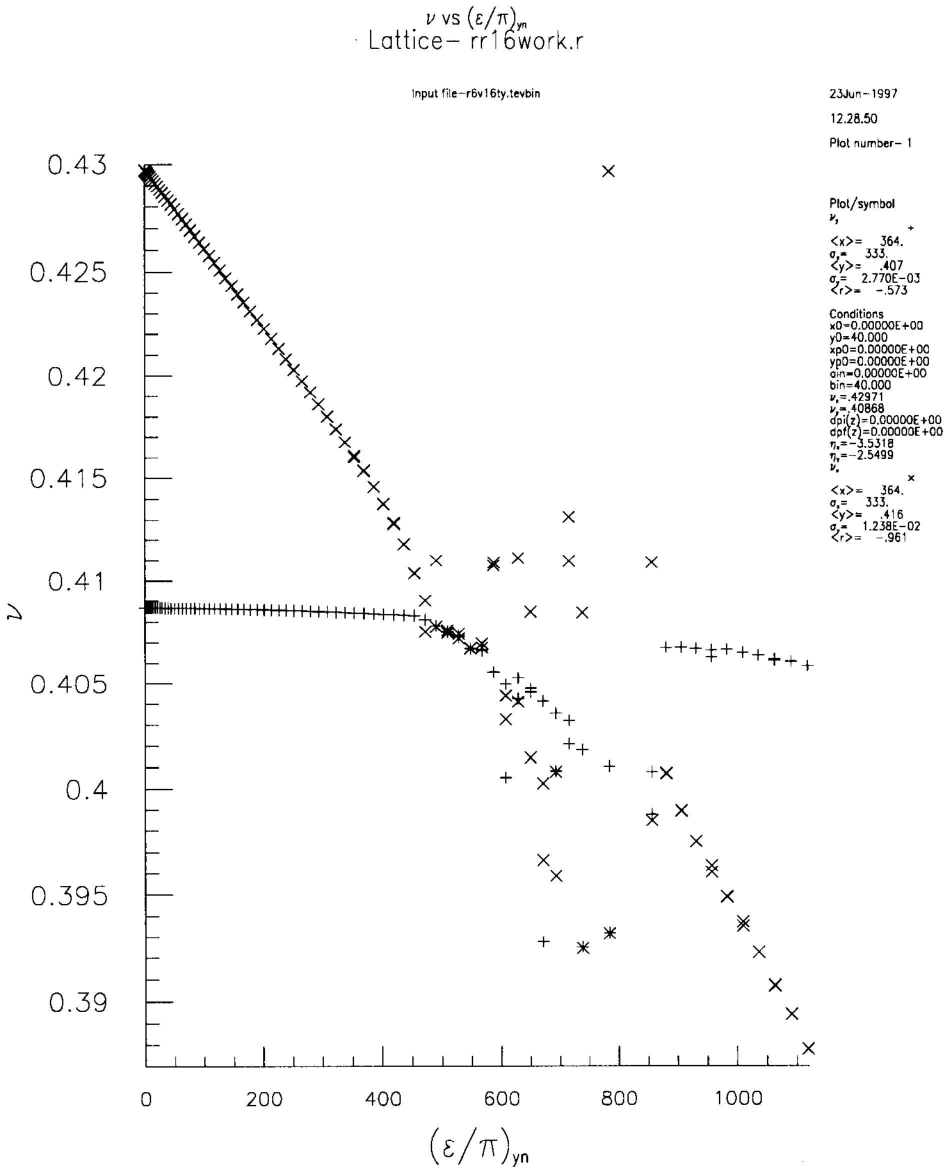

Figure 13: Tune as a function of ϵ_y; Particle started with $y' = x = x' = 0$.

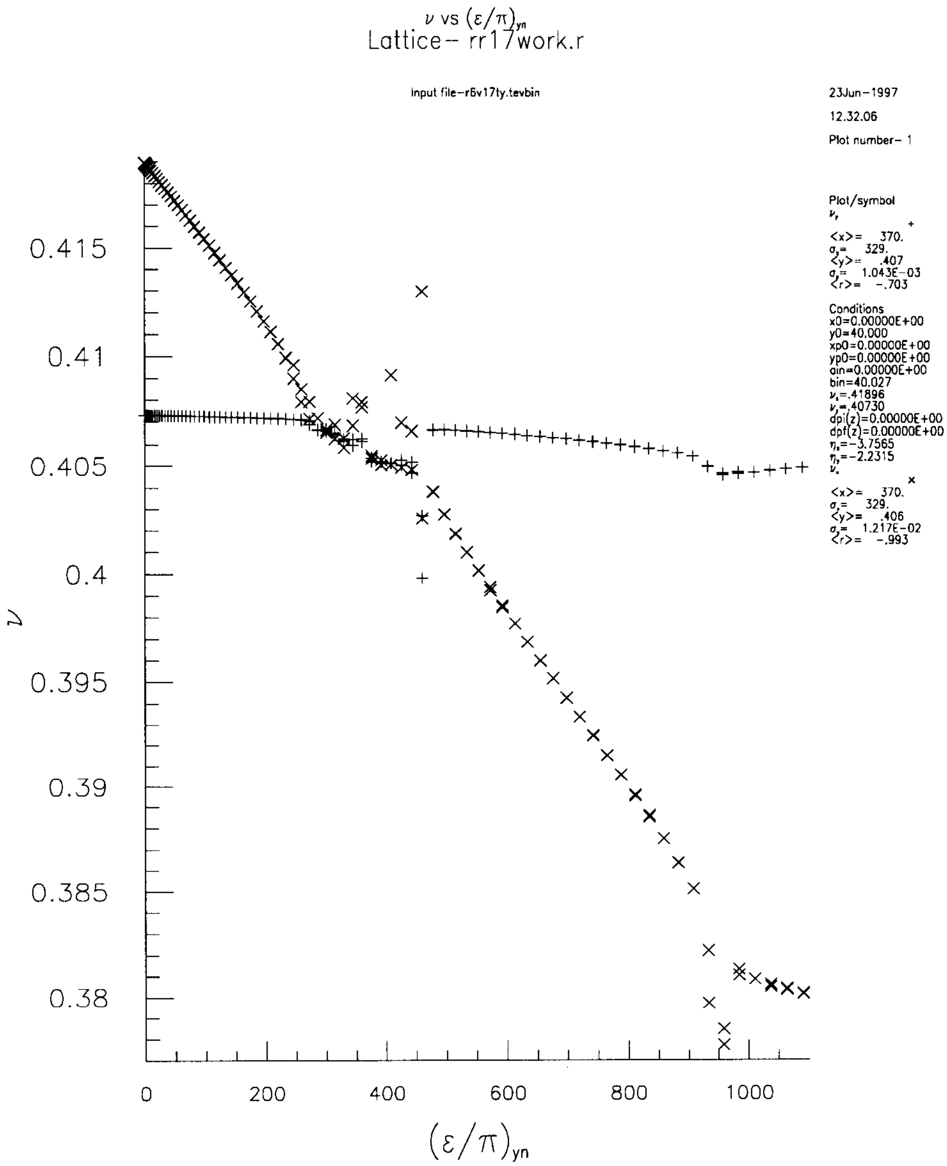

Figure 14: Tune as a function of ϵ_y; Particle started with $y' = x = x' = 0$.

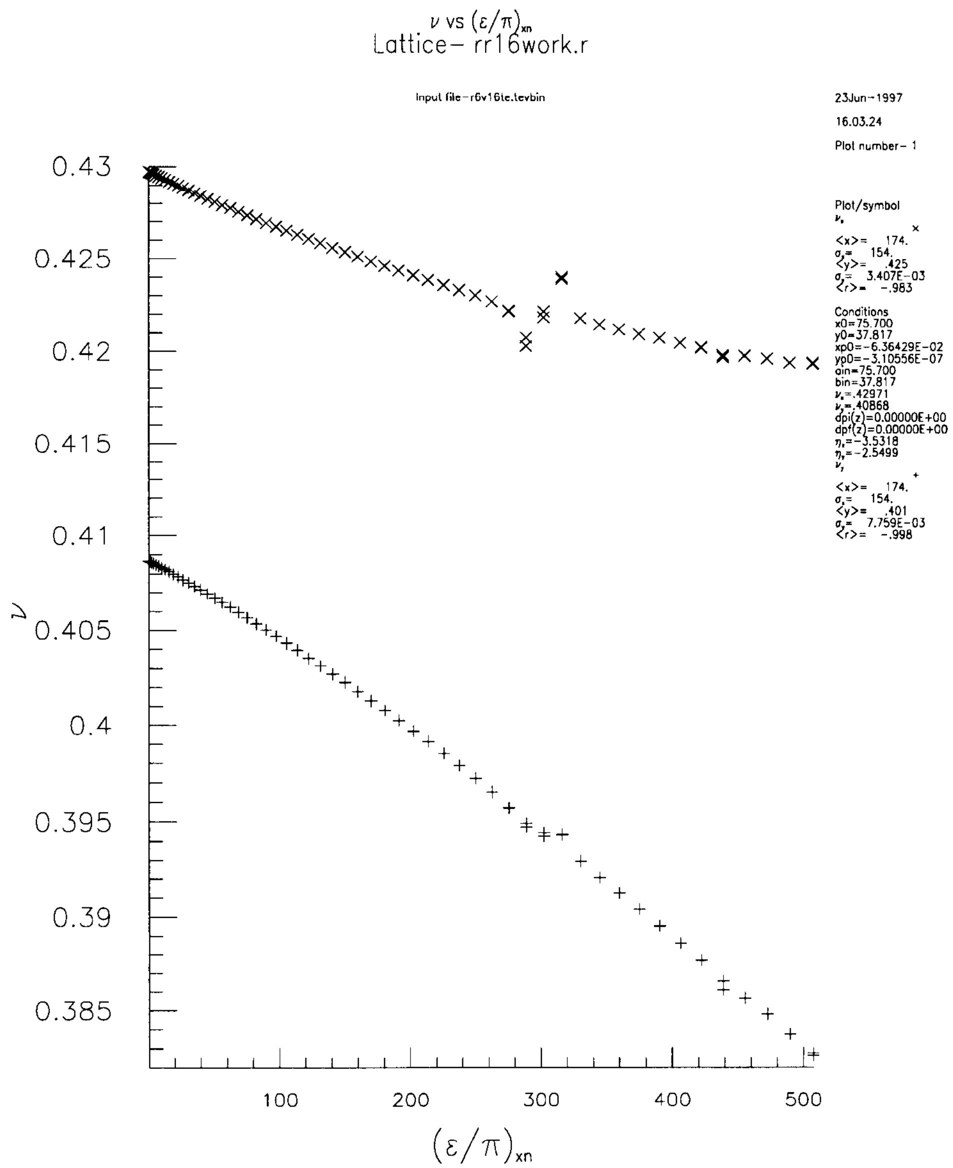

Figure 15: Tune as a function of ϵ_x=ϵ_y; Particle started with $x' = y' = 0$.

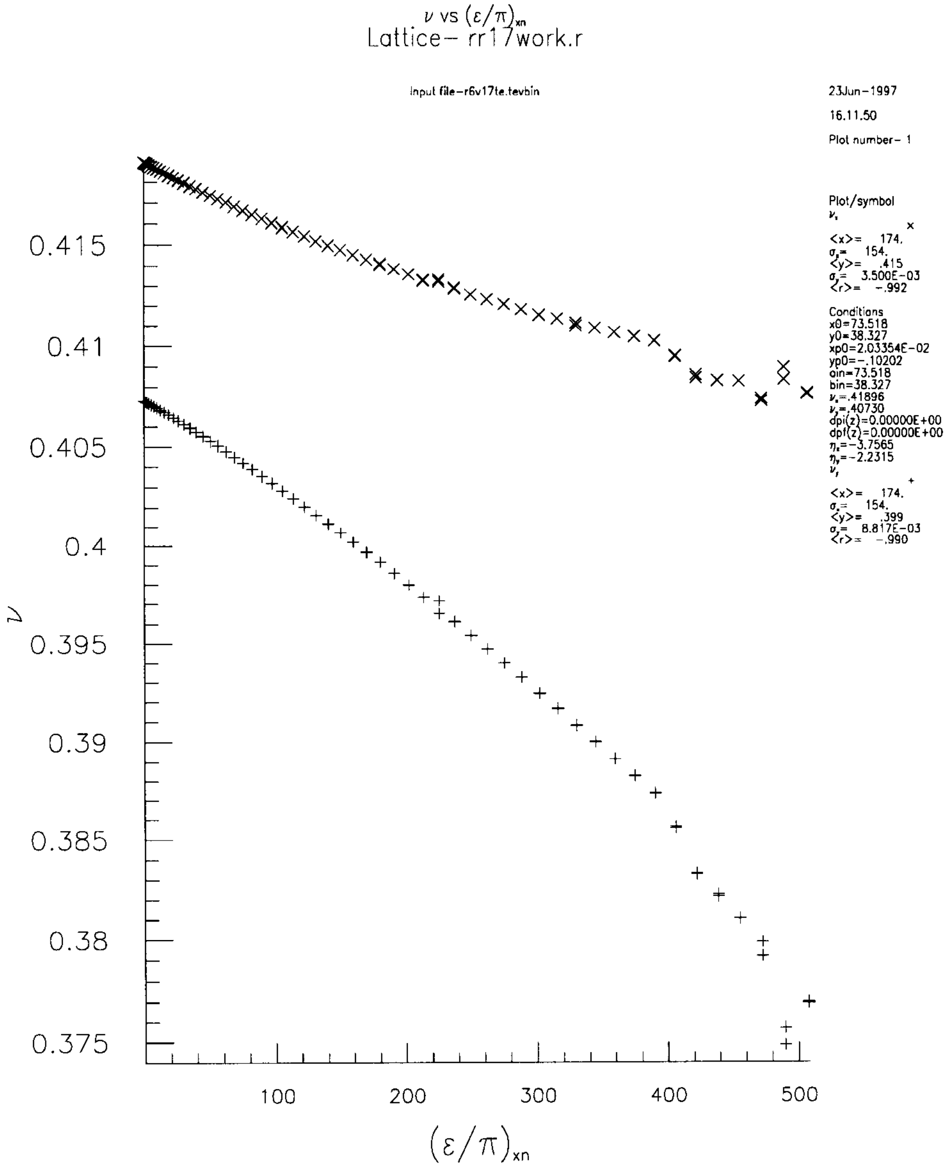

Figure 16: Tune as a function of ϵ_x=ϵ_y; Particle started with $x' = y' = 0$.

It is perhaps interesting to compare the emittance where the amplitude dependence of the tune becomes irregular with (what I regard as the best estimate for) the dynamic aperture based on tracking with more than one initial point on the phase ellipse. In the case of RRv16 the tune dependence is irregular when $\epsilon_x = \epsilon_y \approx 320\pi mmmr$ where as my best estimate for the dynamic aperture is $310\pi mmmr$ (Figure 15). This is reasonable agreement. It will be noted that there is another irregularity at $\epsilon \approx 283\pi mmmr$ (Figures 15 and 17) [9] where the particle is apparently stable. If the number of turns is increased from the 2,048 turns used to determine the dynamic aperture quoted above to 10,240 turns, we discover that there is a region of instability in this region of emittance, and hence the dynamic aperture is now estimated to be less than $283\pi mmmr$.

Similar agreement is seen for RRv17 where I estimate the dynamic aperture to be $\approx 220\pi mmmr$ and an irregularity is seen in the amplitude dependence of the tune at $\epsilon_x = \epsilon_y \approx 200\pi mmmr$. Again there is good agreement.

There is apparently some correlation between the behavior of the amplitude dependence of the tune and particle survival during tracking. If the dependence is *smooth* the particle is stable; if the dependence is *irregular* then starting at some point on the linear phase ellipse the particle is likely to be lost, if we track for enough turns. The nature and origin of the correlation should be investigated to see how this might be used to evaluate the dynamic aperture of a lattice. It may be only an artifact of having only a sextupole non-linearity and might not be of any use when other high order multipoles contribute to the fields in the magnets.

Conclusion

The dynamic aperture cannot be characterized by a single number but instead depends on both ϵ_x and ϵ_y in some complicated manner. For both RRv16 and RRv17 the stability of a particle, for a given emittance, can depend on the initial phase of the particle on the linear phase space ellipse.

Since the dynamic aperture depends on how we do the tracking, the number of turns, the number of values on the linear phase ellipse at which we begin the tracking, and the size of the step with which we vary the emittance (we can jump over unstable regions if the step size is to large), the difference

[9] The difference between Figures 15 and 17 is the size of the step that I use to vary the emittance.

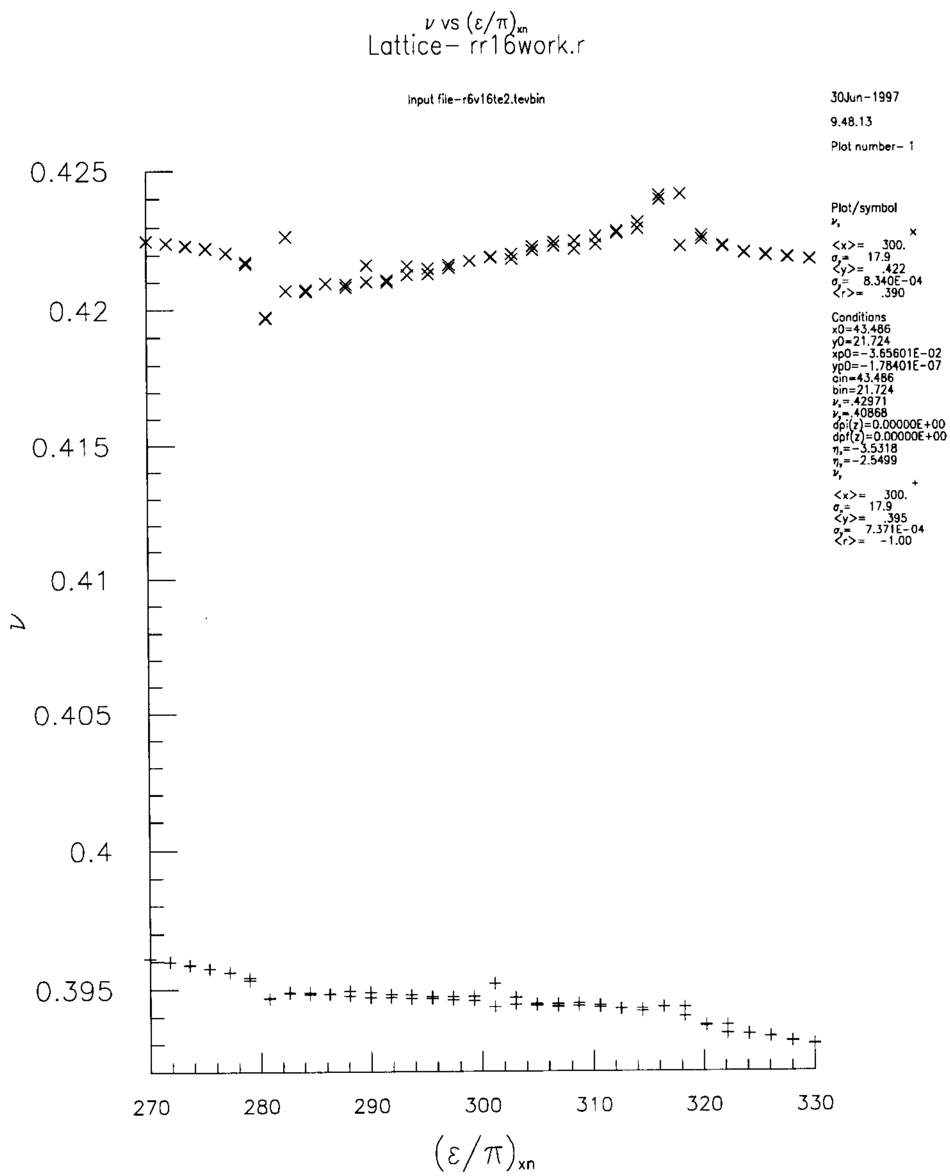

Figure 17: Tune as a function of ϵ_x=ϵ_y; Particle started with $x' = y' = 0$.

in the dynamic aperture between RRv16 and RRv17 may not be real and may simply reflect the conditions used to determine the dynamic aperture.

The amplitude dependence of the tune shift shows *irregular* behavior at the values of the amplitude where tracking results show particle instability. This may turn out to be a useful tool in looking for regions of instability.

Coda

As I read this note I am struck by how disconnected it seems and how few conclusions I felt capable of reaching. On the other hand I don't feel that the results are inferior to other tracking studies of which I am aware. I am convinced that after more than 15 years, we still do not know how to understand the data that is generated by following the trajectory of a particle in phase space and how it relates to the behavior of a beam in an accelerator. Of course near a resonance accelerator models have been successful in predicting the dynamic aperture of the lattice. On the other hand the calculations have been, I believe, much less successful at the actual operating points of the same accelerators.

I am convinced that instead of tracking, as the computing power increases, for an ever increasing number of turns, and with a larger number of seeds, it would be more useful, that we try and better understand what it is that limits the stable phase space in a highly non-linear lattice. First order driving terms, that is first order in the strength of the non-linear elements, are of little value in a lattice like the Recycler, where the amplitude dependent tune shift depends on the fourth power of the initial displacement, for values of the displacement near the stable limit and on the fourth power of the strength of the sextupole fields[10].

We should look at the tracking results as data for our analysis not as its conclusions.

[10] I hope to discuss these points in a subsequent note